Solving Angles in a Parallelogram

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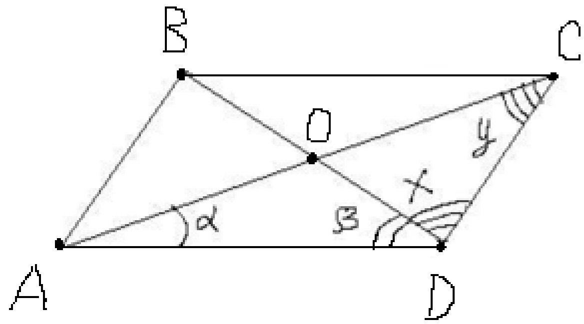
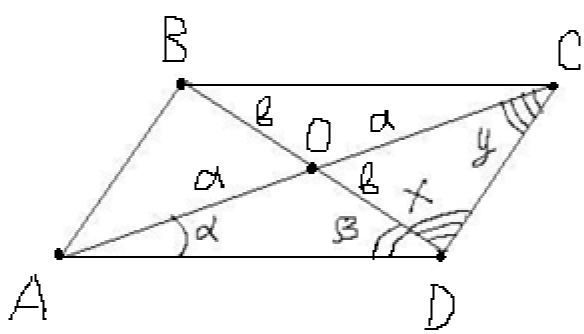
December 2017



Triangle AOD uniquely defines all other parts of parallelogram ABCD: the opposite triangle BOC (which is congruent to AOD) and two adjacent triangles, AOB and DOC (which are conguent to each other). In particular, angles α and β, uniquely define all other angles in parallelogram ABCD, including angles x and y.

If we manage to express the measure of angle x in terms of the measures of angles α and β, the measures of all other angles in parallelogram ABCD can be easily calculated.

We can use the Law of Sines in trianles AOD and DOC to find the formula expressing the measure of angle x in terms of the measures of angles α and β.



$\frac{\text{Sin x}}{\text{Sin y}} = \frac{a}{b}$; $\frac{\text{Sin}\,\beta}{\text{Sin}\,\alpha} = \frac{a}{b}$; $\frac{\text{Sin x}}{\text{Sin y}} = \frac{\text{Sin}\,\beta}{\text{Sin}\,\alpha}$; Angle DOC = α+β;

y = 180° - (α + β) – x; $\frac{\text{Sin x}}{\text{Sin}\,(180° - (\alpha+\beta) - x)} = \frac{\text{Sin}\,\beta}{\text{Sin}\,\alpha}$; $\frac{\text{Sin x}}{\text{Sin}\,(\alpha+\beta+x)} = \frac{\text{Sin}\,\beta}{\text{Sin}\,\alpha}$;

$$\text{Sin } x = \frac{\text{Sin } \beta}{\text{Sin } \alpha} \cdot \text{Sin } (\alpha + \beta + x) \ ;$$

$$\text{Sin } x = \frac{\text{Sin } \beta}{\text{Sin } \alpha} \cdot \text{Sin } (\alpha + \beta) \cdot \text{Cos } x + \frac{\text{Sin } \beta}{\text{Sin } \alpha} \cdot \text{Cos } (\alpha + \beta) \cdot \text{Sin } x;$$

$$\text{Sin } x \cdot \left(1 - \frac{\text{Sin } \beta}{\text{Sin } \alpha} \cdot \text{Cos } (\alpha + \beta)\right) = \frac{\text{Sin } \beta}{\text{Sin } \alpha} \cdot \text{Sin } (\alpha + \beta) \cdot \text{Cos } x;$$

$$\text{Tan } x = \frac{\text{Sin } \beta \cdot \text{Sin } (\alpha + \beta)}{\text{Sin } \alpha - \text{Sin } \beta \cdot \text{Cos } (\alpha + \beta)} \ ;$$

$$x = \arctan \left(\frac{\text{Sin } \beta \cdot \text{Sin } (\alpha + \beta)}{\text{Sin } \alpha - \text{Sin } \beta \cdot \text{Cos } (\alpha + \beta)} \right) .$$